Fair Shares: Feasibility, Domination and Incentives

Moshe Babaioff and Uriel Feige

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Fair Partition of Indivisible Goods

- We consider **share-based** approach to fair partition of indivisible goods among equally-entitled agents (without monetary transfers).

Definition: A share s is a function that maps (v_i, n) to non-negative value $s(v_i, n)$ satisfying "independence of item names" and is "realizable".

- A share is feasible if for any valuations we can always give every agent her share.
- The maximin share (MMS) is not feasible for additive valuations.
- For a share to serve as a fairness **guarantee**, it must be feasible.

Self-Maximizing Shares

Bundle B is acceptable to i if and only if $v_i(B) \geq s(v_i, n)$

The share guarantee is the value of the lowest acceptable bundle:

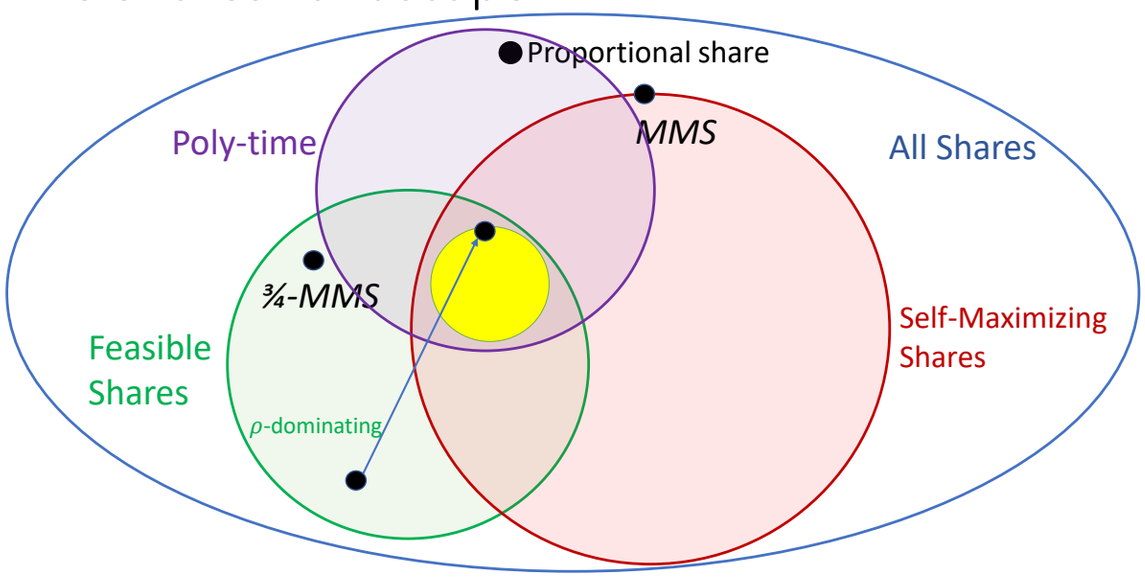
$$\hat{s}(v_i, n) = \min\{v_i(B) \mid B \text{ is acceptable to } i\}$$

Definition: A share is self-maximizing if truthful reporting maximizes the share guarantee (with respect to the truth!).

Theorem: Let C be any class of valuations that contains all additive valuations over goods. For C there is no feasible share that dominates all feasible shares that are self maximizing.

Theorem: Let C be an arbitrary class of valuations; and let s be an arbitrary feasible share for class C. Then for class C there exists a feasible share s' that dominates s and is self-maximizing.

The Shares Landscape



ρ-dominating and poly-time SM feasible shares

- Theorem: For the class of additive valuations with n agents, the $NS_{n,3}$ share is feasible, self-maximizing, polynomial-time computable and $\frac{2n}{3n-1}$ −dominating.
  For at most four agents this share is $\frac{4}{5}$ −dominating.
- Furthermore, for two agents, for any positive ϵ there exists a share that is feasible, self-maximizing, polynomial-time computable and is $(1-\epsilon)$ −dominating.