Quantal Correlated Equilibrium
in Normal Form Games

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"Coordinating subrational players using a signaling device"

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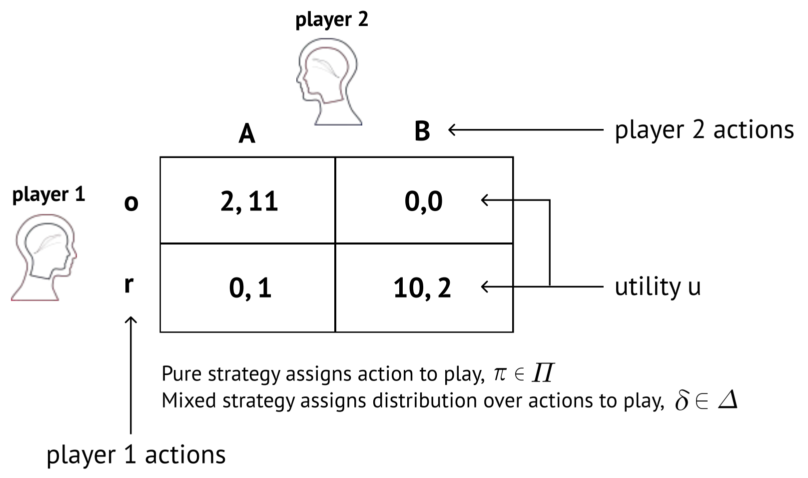
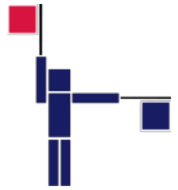
Game model

Normal Form Games



Correlation Device



Each player has a set of possible signals *S*

Signals are sent privately

Distribution λ over signal tuples is a public knowledge

Model of subrationality

Quantal Response

Suggests that players take suboptimal actions with non-zero probability

Actions with higher expected utility are chosen with higher probability

Formally defined using a generalized Luce model:

Given a continuous, increasing function *q*, every player plays the following quantal distribution over actions:

$$QR_i(\delta_{-i}) = \left(\frac{q_i(u_i(\delta_{-i}, a_i))}{\sum_{a'_i \in A_i} q_i(u_i(\delta_{-i}, a'_i))} \right)_{a_i \in A_i}$$

Solution concept

Quantal Correlated Equilibrium (QCE)

All players quantal-respond after receiving their private signals

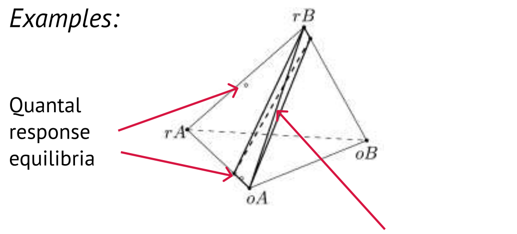
$$u_i(a_i|s_i) = \sum_{a_{-i} \in A_{-i}} \sum_{s_{-i} \in S_{-i}} \lambda(s_i, s_{-i}) \delta_{-i}(a_{-i}|s_{-i}) u_i(a_i, a_{-i})$$

$$\delta_i(a_i|s_i) = \frac{q_i(u_i(a_i|s_i))}{\sum_{a'_i \in A_i} q_i(u_i(a'_i|s_i))}$$

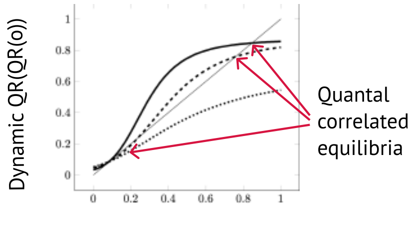
Optimalization with criterion function *f*

$$\max_{\lambda \in \Lambda, \sigma \in \Sigma} f(\lambda, \sigma) \qquad s.t. \;\; \sigma \in QCE(\lambda)$$

Examples:



Polytope of correlated equilibria



Probability of action o

Properties of Quantal Correlated Equilibrium

Relations: (1) QCE is an agent quantal response equilibrium in the extended game.
(2) Any quantal response equilibrium may be extended into a QCE.
(3) Let *Q* be a sequence of quantal response functions that approach the best response in the infinity. Then the limit quantal correlated equilibrium is a correlated equilibrium.

Advantage of signaling: Let q_i be exponential for each player and *u≥0*. Assume the signaler's utility is negatively correlated with other players' utilities. Then $u_s(QCE) > u_s(QRE)$.

Topology: Let *C = {(λ, QCE(λ))}*. Then *C* is compact and the correspondence λ → QCE(λ) is upper hemicontinuous. If QCE(λ) are unique then *C* is connected.

Complexity: The problem of computing a quantal correlated equilibrium is PPAD-hard.

Applications of Quantal Correlated Equilibrium

1. Content accuracy signaling (anti-missinformation)
2. Cyber-security (network defenses)
3. Trading coordination (supply chains)
4. Robust system design

& others

Computation

Homotopy Formulation of Quantal Correlated Equilibrium

Tracing the path from uniform strategies to QCE using the system:

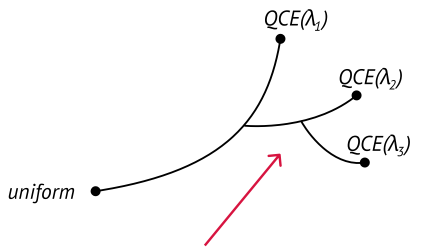
$$H(\delta, t) = \left(H_i^{k,l}(\delta, t) \right)_{i \in N, \;\; a_i^k \in A_i, \;\; s_i^l \in S_i}$$

$$H_i^{k,l}(\delta, t) = f_2(f_1(\hat{q}_i(u_i(a_i^0|s_i^l), t)), \delta_i(a_i^k|s_i^l)) - f_2(f_1(\hat{q}_i(u_i(a_i^k|s_i^l), t)), \delta_i(a_i^0|s_i^l))$$

$$H_i^{0,l}(\delta, t) = \sum_{a_i^k \in A_i} f_1^{-1}(\delta_i(a_i^k|s_i^l)) - 1$$

$$\text{s.t. } f(xy) = f_2(f_1(x), f_1(y)) \quad \forall x, y \in \mathbb{R}$$

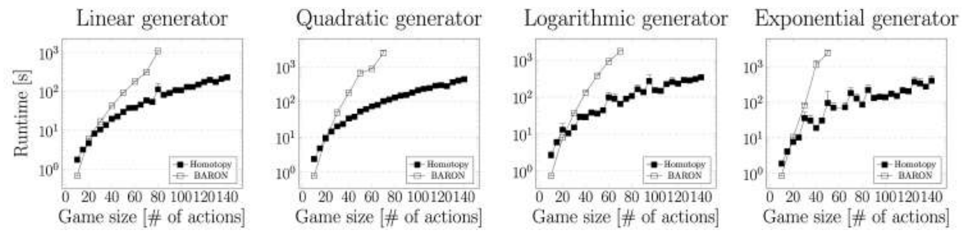
Corrections with the Gauss-Newton method



In each step of tracing we shift the homotopy towards the maximum of criterion *f*

$$\lambda^{\iota+1} \leftarrow P_\Lambda\left(\lambda^\iota + \eta f'(\lambda, \delta_p(\lambda))\right)$$

Comparison to BARON - Algorithm for General Optimization



Homotopy algorithm is faster

#a	Linear generator H [s]	Linear generator B [s]	Linear generator Δ	Quadratic generator H [s]	Quadratic generator B [s]	Quadratic generator Δ	Logarithmic generator H [s]	Logarithmic generator B [s]	Logarithmic generator Δ	Exponential generator H [s]	Exponential generator B [s]	Exponential generator Δ
2	2±0	0±0	$2 \cdot 10^2$	3±0	0±0	$-3 \cdot 10^7$	6±1	0±0	$-2 \cdot 10^6$	1±0	1±0	$-3 \cdot 10^3$
3	7±1	0±0	$1 \cdot 10^2$	7±0	9±1	$-2 \cdot 10^7$	14±4	2±0	$-1 \cdot 10^6$	3±0	51±13	$-1 \cdot 10^3$
4	15±4	0±0	$6 \cdot 10^4$	12±2	1266±472	$4 \cdot 10^7$	27±6	13±6	$-9 \cdot 10^7$	17±13	1191±466	$-7 \cdot 10^5$
5	16±3	7±2	$2 \cdot 10^3$	-	-	-	44±11	45±15	$-2 \cdot 10^3$	-	-	-
6	35±9	169±139	$2 \cdot 10^3$	-	-	-	61±12	159±50	$-8 \cdot 10^7$	-	-	-
7	126±85	267±204	$2 \cdot 10^3$	-	-	-	75±15	1156±422	$-4 \cdot 10^6$	-	-	-

... and provides better solutions

Jakub is looking for a postdoc position!

13 publications – EC, AAAI, IJCAI, etc

See his other work at j-cerny.github.io

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