# Improved Online Contention Resolution for Matchings and Applications to the Gig Economy

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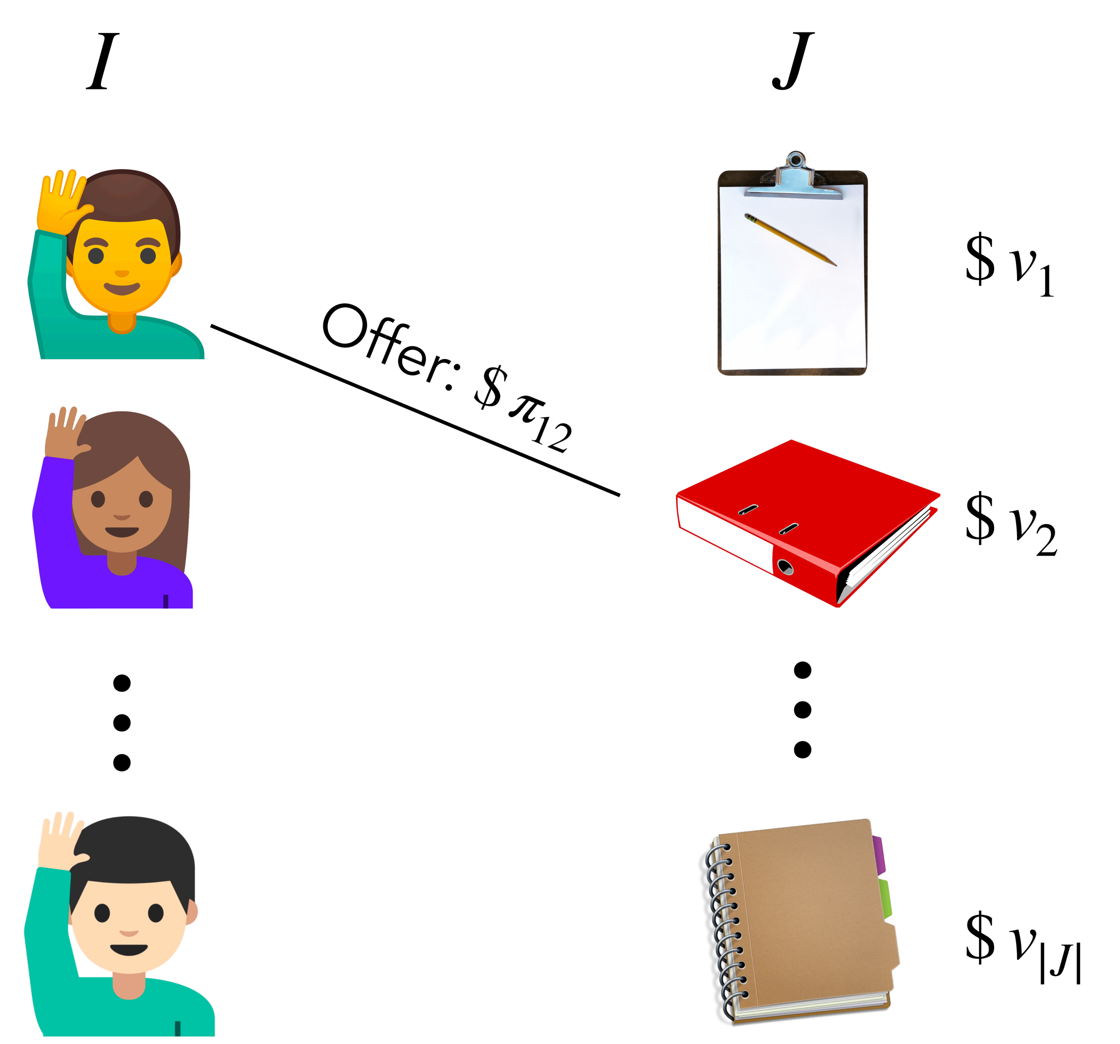
## Motivation: Sequential Pricing Problem




- Individuals $I$, Jobs $J$
- Value $v_j$ for completing job $j \in J$
- Platform can offer a price $\pi_{ij} = w$ for $i$ to do job $j$
  - $i$ accepts with known probability $p_{ijw}$, in which case $i$ and $j$ are permanently matched

**Objective.** Maximize social welfare or revenue.

- NP-hard even for a single item [4], so we study efficient *approximations* to the optimal online policy

## Online Contention Resolution Schemes (OCRS)

**OCRS [3].** Given $G = (V, E)$, $\{x_e\}_{e \in E}$ in fractional matching polytope

- Edges $e \in E$ arrive in some order and are **active** with probability $x_e$
- May match active edges upon arrival

**$c$-balanced OCRS.** $\Pr[\text{match } e] \geq c \cdot x_e$

## Our Results

**Reduction to OCRS.** $c$-approximate pricing mechanism reduces to $c$-balanced random-order OCRS (RO-OCRS) for matching

**New RO-OCRS.** Improved 0.456--balanced RO-OCRS for bipartite matching.

**Corollaries.** Improvements for the *correlation gap* of matching [2], and the well-studied *stochastic matching with patience* problem [1].

## Reduction to OCRS

**LP Relaxation.**

$$\max \sum_{w, e=(ij)} y_{ew} \cdot p_{ew} \cdot (v_j - w) \qquad \textbf{(LP-Pricing)}$$

$$\text{s.t.} \sum_{w} y_{ew} \leq 1 \qquad \forall e \qquad (1)$$

$$\sum_{e \ni v} \sum_{w} y_{ew} \cdot p_{ew} \leq 1 \qquad \forall v \qquad (2)$$

$$y_{ew} \geq 0 \qquad \forall e, w. \qquad (3)$$

**Rounding LP-Pricing (sketch).**

- For every edge $e$, set price $\pi_e$ so $\mathbb{P}[\pi_e = w] = y_{ew}$.
- Query along $e$ with price $\pi_e \Rightarrow e$ successful with probability $x_e := \sum_w y_{ew} p_{ew}$
- $\{x_e\}$ in fractional matching polytope $\Rightarrow$ run RO-OCRS

## Template for Random-Order OCRS (e.g., [1])

**For each** edge $e$, sample a random arrival time $t_e \sim \text{Unif}[0,1]$

**For each** $e$, in increasing order of $t_e$:

**If** $e$ is active and its endpoints are free:

Match $e$ with probability $s(e)$

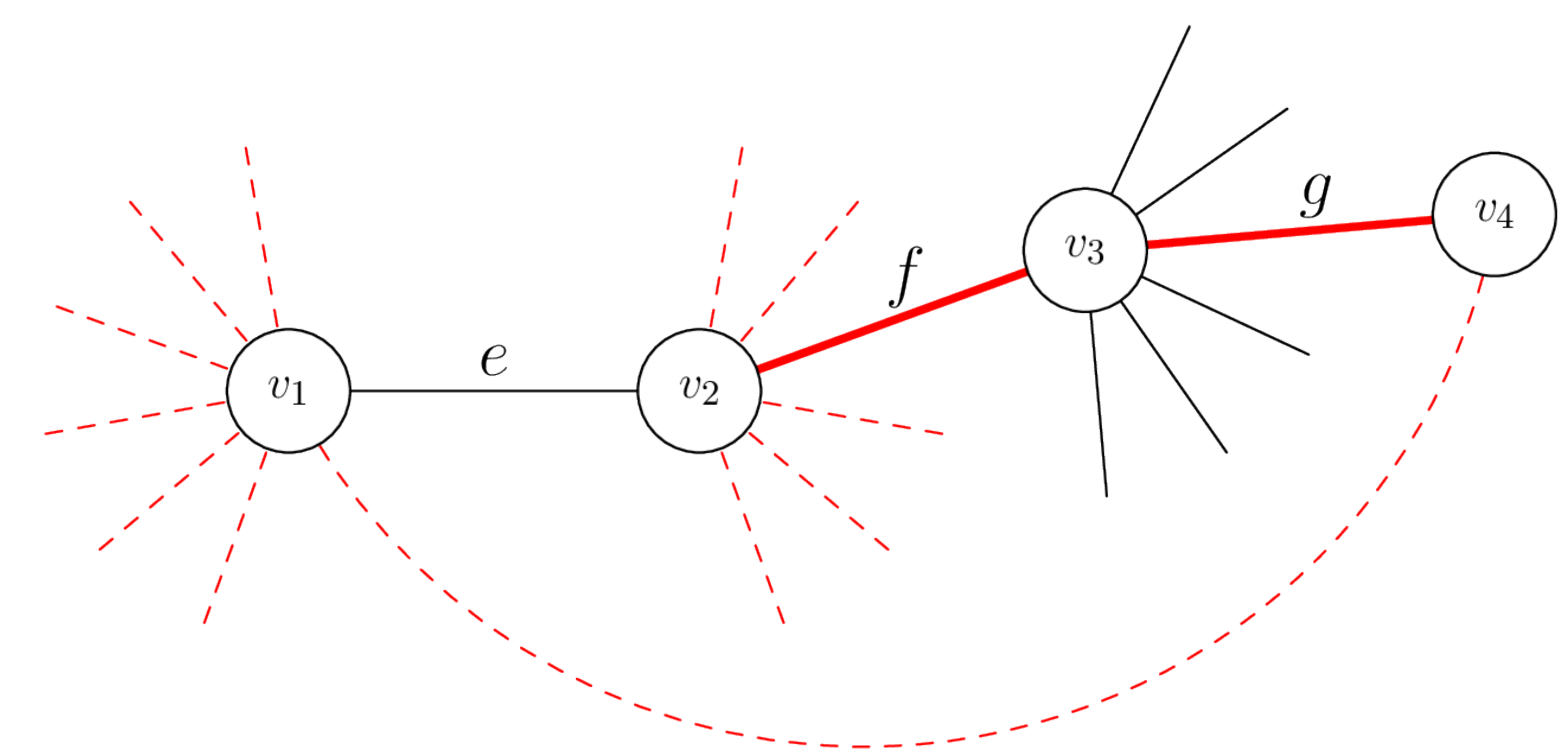
*Downsampling* function for $e$ (function of $\{x_e\}, \{t_e\}$)

## Our Improved Downsampling

**New Downsampling.** $s(e) := \mathrm{e}^{-t_e x_e}(1 - \alpha(2 - x_e - d_e))$ for a parameter $\alpha \in [0, 0.5]$, where $d_e := \sum_{f \sim e} x_f$.

**Analysis idea: Two extremes.**

- If "most" edges $f \sim e$ have $d_f$ significantly less than $2 - x_f$, it is easier to match $e$, as neighbors are downsampled aggressively.
- If "most" edges $f \sim e$ have $d_f \approx 2 - x_f$, then $f$ has a good chance of being blocked by an active $g$, making it easier to match $e$.




- Choose $\alpha$ to balance between these cases

## References

[1] Brian Brubach, Nathaniel Grammel, Will Ma, and Aravind Srinivasan.Improved Guarantees for Offline Stochastic Matching via new Ordered Contention Resolution Schemes. *NeurIPS* (2021).
[2] Simon Bruggman and Rico Zenklusen. An optimal monotone contention resolution scheme for bipartite matchings via a polyhedral viewpoint. *Mathematical Programming* (2020).
[3] Moran Feldman, Ola Svensson, and Rico Zenklusen. Online contention resolution schemes. *SODA* (2016).
[4] Tao Xiao, Zhengyang Liu, and Wenhan Huang. On the complexity of sequential posted pricing. *AAMAS* (2020)